The Euler's Theorem and Product Exhaustion Problem

As soon as it was propounded that the factors of production are paid equal to their marginal products, a difficult problem cropped up over which there has been a serious debate among the famous economists. The difficult problem which has been posed is that if all factors were paid rewards equal to their marginal products, would the total product be just exactly exhausted?

In other words, if each factor is rewarded equal to its marginal product, the total product should be disposed of without any surplus or deficit. The problem of proving that the total production will be just exhausted if all factors are paid rewards equal to their marginal products has been called "Adding- up Problem" or Product Exhaustion Problem.

The two solutions to the problem of product exhaustion have been put forward. First, important solution was put forward by P.H. Wicksteed who assumed the operation of constant returns to scale in production (that is, the first degree homogenous production function) and applied Euler theory to prove the product exhaustion problem.

The second important solution has been provided by J.R. Hicks and RA. Samuleson who used perfect competition model of determination of product and factor prices to prove the product exhaustion problem. We discuss below these solutions of product exhaustion problem.

Wicksteed's Solution of Product Exhaustion Problem with Euler's Theorem:

Philip Wicksteed was one of the first economists who posed this problem and provided a solution for it. Wicksteed applied a mathematical proposition called Euler's Theorem to prove that the total product will be just exhausted if all the factors are paid equal to their marginal products.

Let Q stand for the total output of the product, a stands for the factor labour and b stands for the factor capital and c stands for land. Assuming that there are only three factors employed for production. Then, the adding up problem implies that,

$Q = MP_a \text{ x } a + MP_a \text{ X } b + MP_c \text{ x } c$

That is, the marginal product of factor a multiplied by the amount of factor a plus the marginal product of factor b multiplied by the amount of factor b plus the marginal product of factor c multiplied by the amount of factor c equals the total product of the firm. Marginal products of various factors can be expressed as partial derivatives. Thus, the marginal product of labour (i.e. factor a) can be expressed as ∂W/∂a, and the marginal product of capital (factor b) as ∂W/∂b, and the marginal product of land (factor c) as ∂W/∂c, then for the adding-up problem (i.e. product exhaustion problem) to be fulfilled, the following equation must hold good:

$$Q = a\frac{\partial Q}{\partial a} + b\frac{\partial Q}{\partial b} + c\frac{\partial Q}{\partial c},$$

where $a\frac{\partial Q}{\partial a}$ represents share of the total product going to labour.

$a\frac{\partial Q}{\partial b}$ represents share of the toal product going to capital

$a\frac{\partial Q}{\partial c}$ represents share of the total product going to land.

Now, Euler's Theorem states that if production function is a homogenous function of the first degree, that is, if in Q =f (a, b, c) for any increase in the variables a, b and c by the amount n, the output Q also increases by n, then Q will be equal to the total sum of the partial derivatives of production function with respect to various factors multiplied by the amounts of the factors respectively.

The homogeneous function of the first degree or linear homogeneous function is written in the following form:

nQ = f(na, nb, nc)

Now, according to Euler's theorem, for this linear homogeneous function:

$$Q = a\frac{\partial Q}{\partial a} + b\frac{\partial Q}{\partial b} + c\frac{\partial Q}{\partial c}$$

Thus, if production function is homogeneous of the first degree, then according to Euler's theorem the total product is:

$$Q = a\frac{\partial Q}{\partial a} + b\frac{\partial Q}{\partial b} + c\frac{\partial Q}{\partial c}$$

Where Q represents the total product and ∂W/∂a, ∂W/∂b, ∂W/∂c are partial derivatives of the production function and therefore represent the marginal products of labour, capital, and land respectively. It follows therefore that if production function is homogeneous of the first degree (that is, where there are constant returns to scale), then, according to Euler's Theorem, if the various factors a, b and c are paid rewards equal to their marginal products, the total product will be just exhausted, with no surplus or deficit.

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We thus see that Euler's Theorem is able to explain product exhaustion when production function is homogenous of the first degree. In this way, Wicksteed assuming constant returns to scale and applying Euler's Theorem, proved the adding-up problem, that is, demonstrated that if all factors are paid equal to their marginal products, the total product will be just exactly exhausted.

A Critique of Euler's Theorem and Wicksteed's Solution:

Wicksteed's solution was criticized by Walras, Barone, Edgeworth and Pareto. It was asserted by these writers that production function was not homogeneous of the first degree, that is; returns to scale are not constant in the actual world. Thus Edge worth satirically commented on Wicksteed's solution, "There is magnificence in this generalization which recalls the youth of philosophy. Justice is a perfect cube, said the ancient sage; and rational conduct is a homogeneous function, adds the modern savant".

Critics pointed out that production function is such that it yields a U- shaped long-run average cost curve. The U-shape of the long-run average cost curve implies that up to a point increasing returns to scale occur and after it diminishing returns to scale are obtained.

In case a firm is still working under increasing returns to scale, then if all factors are paid equal to their marginal products, the total factor rewards would exceed the total product. On the other hand, if a firm is working under diminishing returns to scale, and if all factors are paid equal to their marginal products the total factor rewards would not fully exhaust the total product and will therefore leave a surplus. It follows that Euler's Theorem does not apply and therefore the adding-up problem does not hold good when either there is increasing returns to scale or decreasing returns to scale.

Another drawback pointed out in Wicksteed's solution is that when there are constant returns to scale, the long -run average cost curve of the firm is a horizontal straight line which is incompatible with perfect competition. (Under horizontal long-run average cost curve, the firm cannot have a determinate equilibrium position). But perfect competition was essential to the marginal productivity theory and therefore to Wicksteed's solution. Thus Wicksteed solution leads us to two contradictory things.

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Wicksell, Walras and Barone's Solution of Production Exhaustion Problem:

After Wicksteed, Wicksell, Walras and Barone, each independently, advanced more satisfactory solution to the problem that marginally determined factor rewards would just exhaust the total product. These authors assumed that the typical production function was not homogeneous of the first degree, but was such that yielded U-shaped long-run average cost curve.

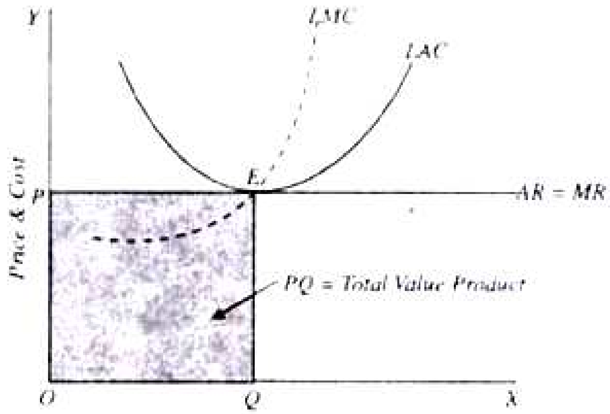
They pointed out that in the long-run under perfect competition the firm was in equilibrium at the minimum point of the long-run average cost curve. At the minimum point of the long-run average cost curve, the returns to sc ale are momentarily constant, that is, returns to scale are constant within the range of small variations of output.

Thus the condition required for the marginally determined rewards to exhaust the total product, that is the operation of constant returns to scale, was fulfilled at the minimum point of the long-run average cost curve, where a perfectly competitive firm is in long-run equilibrium. Thus in the case of perfectly long-run equilibrium, Euler Theorem can be applied and if the factors are paid rewards equal to their marginal products, the total product would be just exactly exhausted.

Hicks-Samuelson's Solution to the Product Exhaustion Problem:

After Wicksell, Walras and Barone, J.R. Hicks and P. A. Samuelson provided more satisfactory-solution to the problem of product exhaustion problem. The basic point to note in their solution is that it is the market conditions of perfect competition with its important feature of zero economic profits in the long run and not the first degree-homogeneous production function that ensures that if factors are paid rewards equal to their marginal products, total value product would be just exhausted.

In a perfectly competitive market structure, firms make neither economic profit nor make losses. Thus the solution of product exhaustion problem in case of the firms working in competitive factor markets where factors are paid equal to their marginal products, the existence of perfect competition in the product markets will ensure zero economic profits in the long run. Consider Figure at next page, where a perfectly competitive firm is in long- run equilibrium at the minimum point of the long-run average cost curve LAC producing level of output OQ at price OP.



The total value product produced by the firm in this long-run equilibrium is equal to the area OPEQ. Since price OP is equal to average cost (AC) at this long-run equilibrium output with zero pure profits, total value product (PQ) will be equal to the total cost (TC). Thus

In long-run competitive equilibrium:

Total Value Product (P.Q.) = w.L+K.r. …(1)

Now marginal productivity theory of distribution requires that

$w = VMP_L = P.MPP_L$ …(2)

$r = VMP_K = P.\ MPP_K$ …(3)

Where w and r are prices of labour and capital respectively and MPP_L and MPP_K are marginal physical products of labour and capital respectively and P is the price of the product.

Substituting the values of w and r into equation (1) we have

$P.Q = L.\ (P.\ MPP_L) + K.\ (P.\ MPP_K)$

Dividing both sides by P we have

$Q = L.MPP_L + K.\ MPP_K$

That is, if labour and capital are paid equal to their marginal physical products, total output will be just exhausted.

It is important to note that in contrast to the solutions of Wicksteed and of Wicksell, Walras and Barone, the solution furnished by Hicks and Samuelson proves the product exhaustion theorem without assuming constant returns to scale (i.e. first-degree-homogeneous production function) and without using Euler theorem. They prove it by just assuming conditions of perfect market structure.

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The merit of the Hicks-Samuleson solution is that it highlights when conditions of perfect competitive market do not hold, that is, when there is either monopoly or imperfect competition in the product market or monopsony or imperfect competition in the factor market, the hired factors do not get rewards equal to the value of their marginal products and are therefore exploited by the entrepreneurs who may enjoy large economic profits.

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